INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 2.17]

SINUSOIDAL MODEL

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

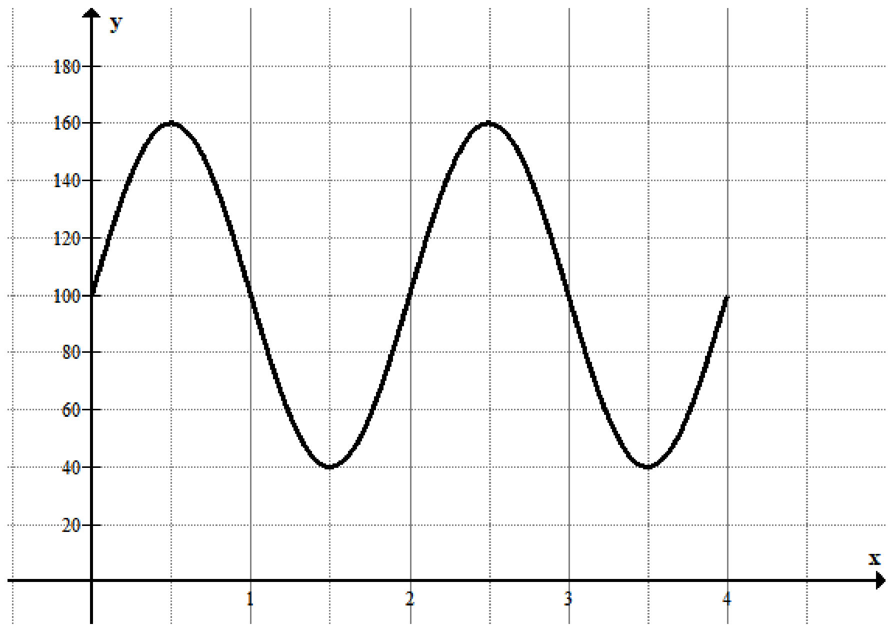
1. [Maximum mark: 30]

Complete the following table

Function	Amplitude	Period	Range
$f(x)=\sin x°$ $f(x)=\cos x°$	1	360	$-1 \le y \le 1$
$f(x)=\sin(10x)°$ $f(x)=\cos(10x)°$			
$f(x)=\sin(36x)°$ $f(x)=\cos(36x)°$			
$f(x)=\sin(180x)°$ $f(x)=\cos(180x)°$			
$f(x)=2\sin(10x)°$ $f(x)=2\cos(10x)°$			
$f(x)=\sin(10x)°+50$ $f(x)=\cos(10x)°+50$			
$f(x)=2\sin(10x)°+50$ $f(x)=2\cos(10x)°+50$			
$f(x)=-2\sin(10x)°$ $f(x)=-2\cos(10x)°$			
$f(x)=-2\sin(10x)°+50$ $f(x)=-2\cos(10x)°+50$			
$f(x)=100-20\sin(10x)°$ $f(x)=100-20\cos(10x)°$			

2. [Maximum mark: 9]

Part of the graph of a trigonometric function $y = f(x)$ is given below. There is a local maximum at $(0.5,160)$ and a local minimum at $(1.5,40)$.



(a) Write down

(i) the equation of the central/principal axis

(ii) the amplitude

(iii) the period [3]

(b) Express the function in the form $y = A\sin(Bx)° + C$ [3]

(c) Find the values of y

(i) when $x = 1.5$ (ii) when $x = 1.6$ [2]

(d) Find the first positive value of x for which $y = 140$. [1]

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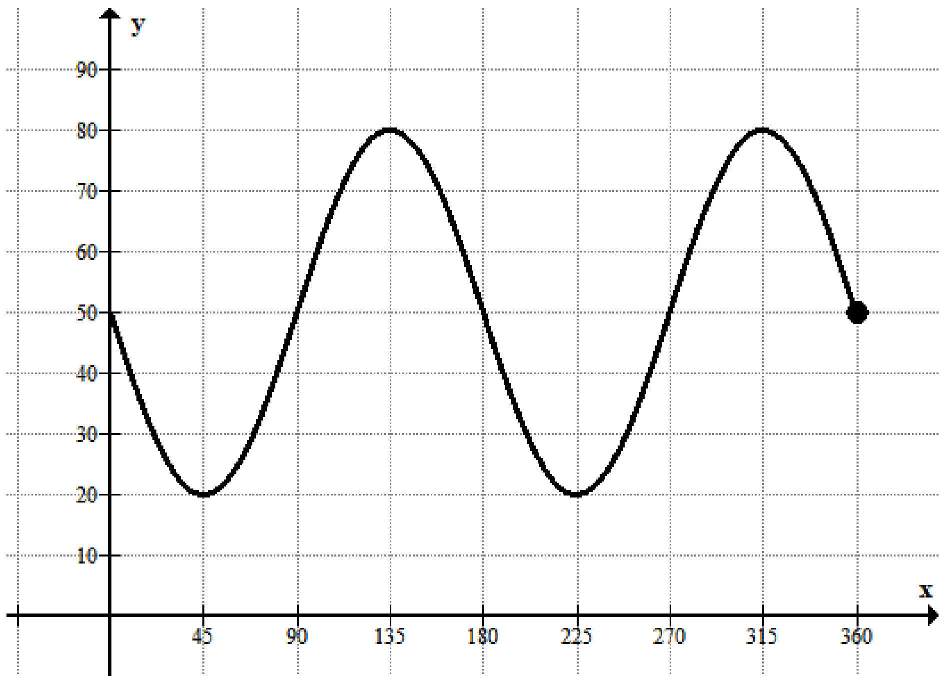
3. [Maximum mark: 9]

The graph of a trigonometric function $y = f(x)$, $0 \le x \le 360$ is given below. There is a local minimum at $(45, 20)$ and a local maximum at $(135, 80)$.



(a) Write down

(i) the equation of the central/principal axis

(ii) the amplitude

(iii) the period [3]

(b) Express the function in the form $f(x) = A\sin(Bx)° + C$ [3]

(c) Solve the equation $f(x) = 30$. [1]

(c) Find the values of x for which $f(x) \le 30$. [2]

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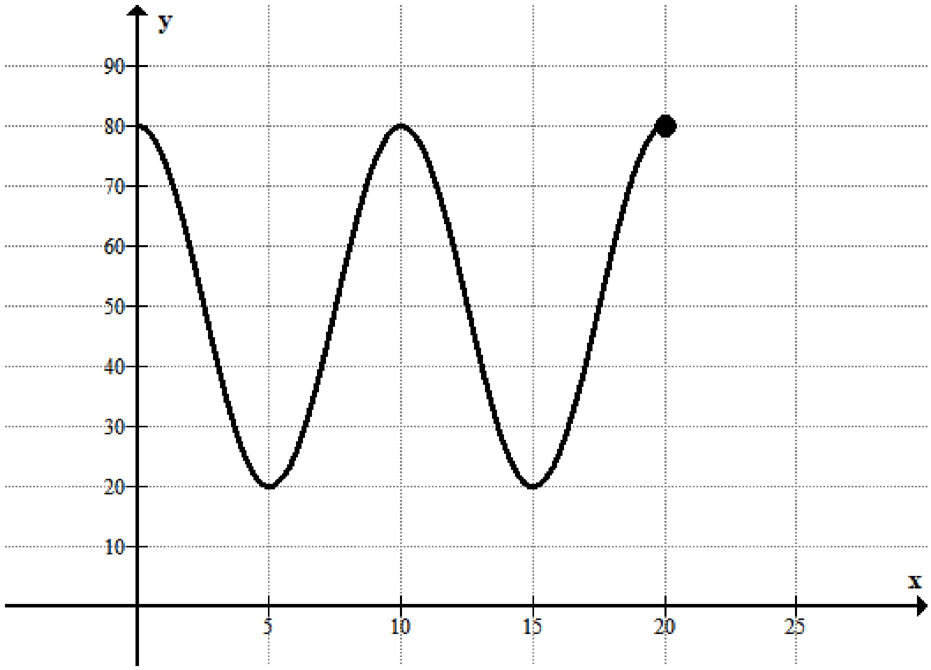
4. [Maximum mark: 10]

The graph of a trigonometric function $y = f(x)$, $0 \le x \le 20$ is given below. There is a local minimum at $(5, 20)$ and a local maximum at $(10, 80)$.



(a) Write down

(i) the equation of the central/principal axis

(ii) the amplitude

(iii) the period [3]

(b) Express the function in the form $f(x) = A\cos(Bx)^{\circ} + C$. [3]

(c) Solve the equation $f(x) = 70$. [2]

(c) Find the values of x for which $f(x) \ge 70$. [2]

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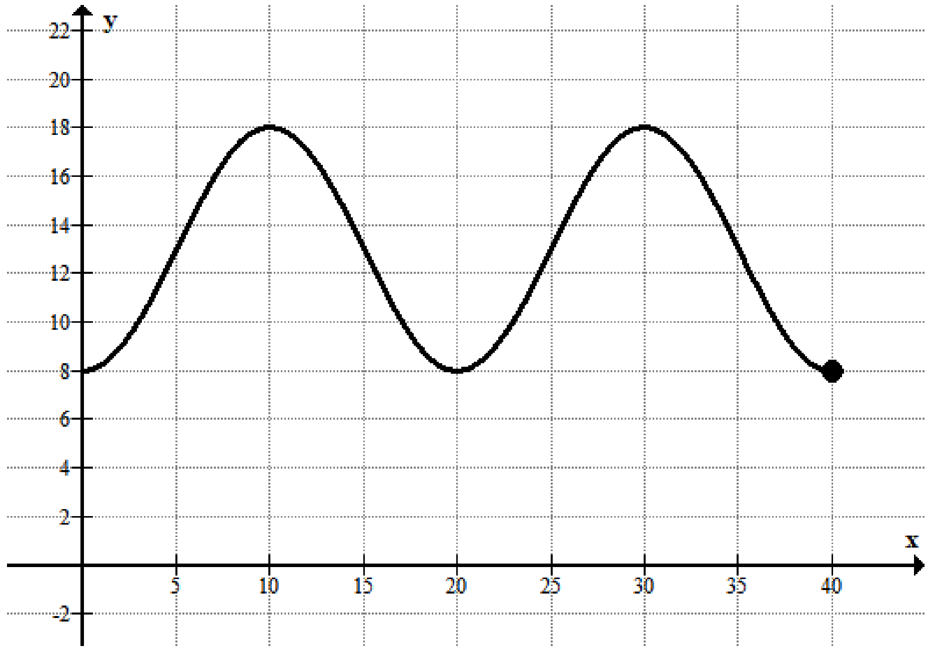
5. [Maximum mark: 8]

The graph of a trigonometric function $y = f(x)$, $0 \le x \le 40$ is given below. There is a local maximum at $(10, 18)$ and a local minimum at $(20, 8)$.



(a) Write down

(i) the equation of the central/principal axis

(ii) the amplitude

(iii) the period [3]

(b) Express the function in the form $f(x) = A\cos(Bx)° + C$ [3]

(c) Solve the equation $f(x) = 8$. [2]

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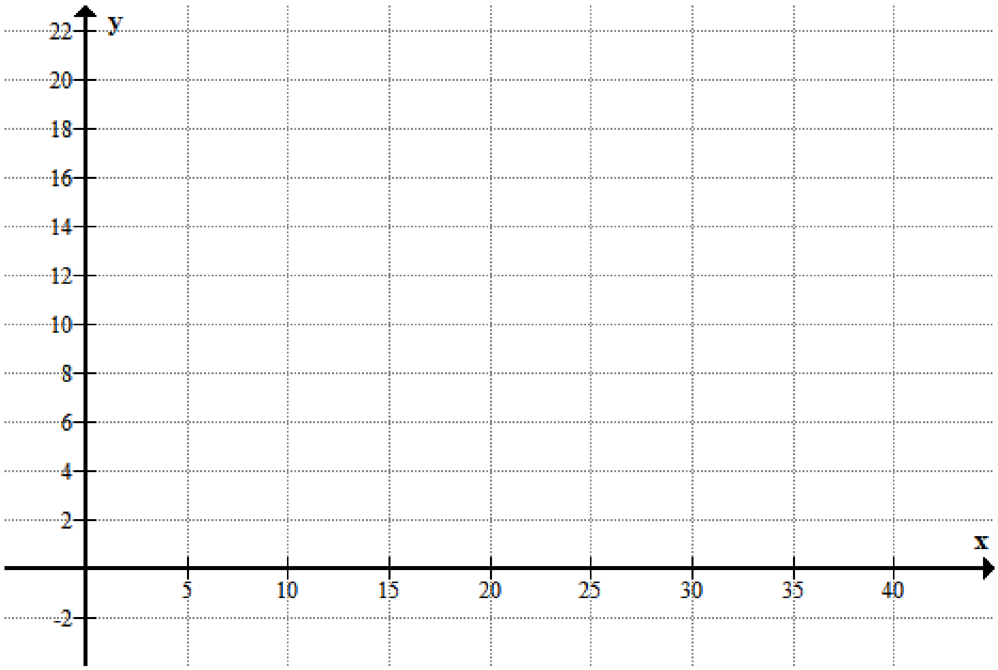
6. [Maximum mark: 4]

Sketch the graph of the function

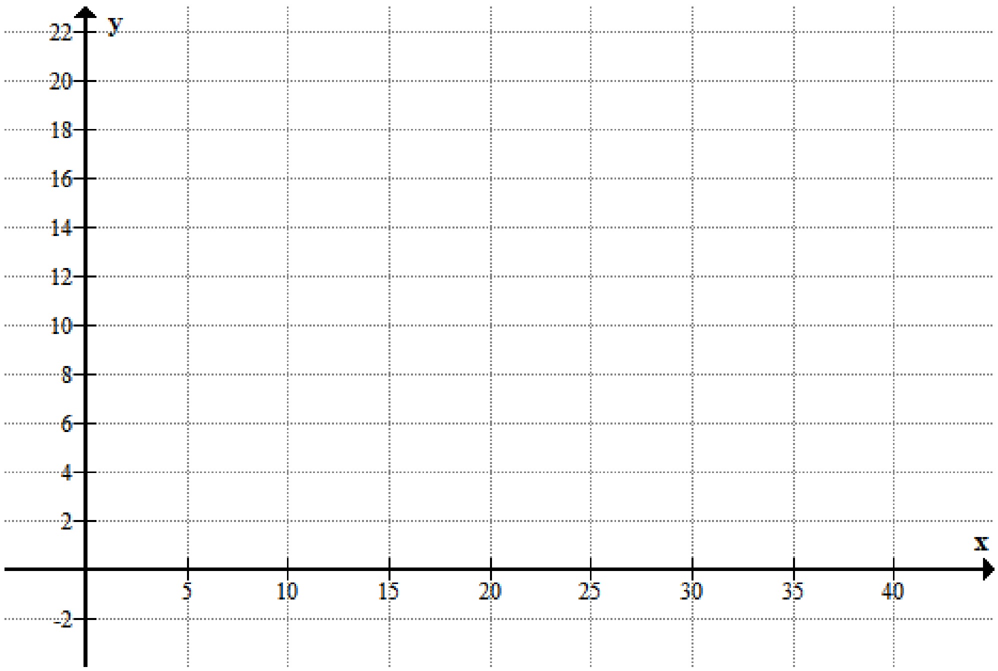
$$f(x) = 6\sin(18x)° + 10, \quad 0 \le x \le 40$$



7. [Maximum mark: 4]

Sketch the graph of the function

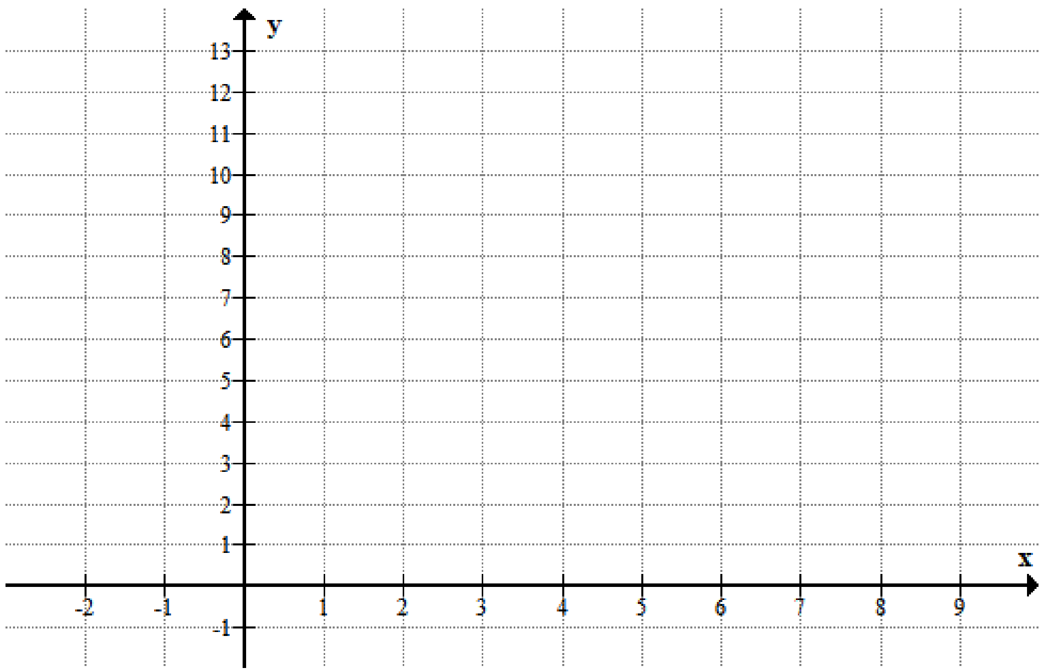
$$f(x) = -8\sin(18x)° + 10, \quad 0 \le x \le 40$$



8. [Maximum mark: 4]

Sketch the graph of the function

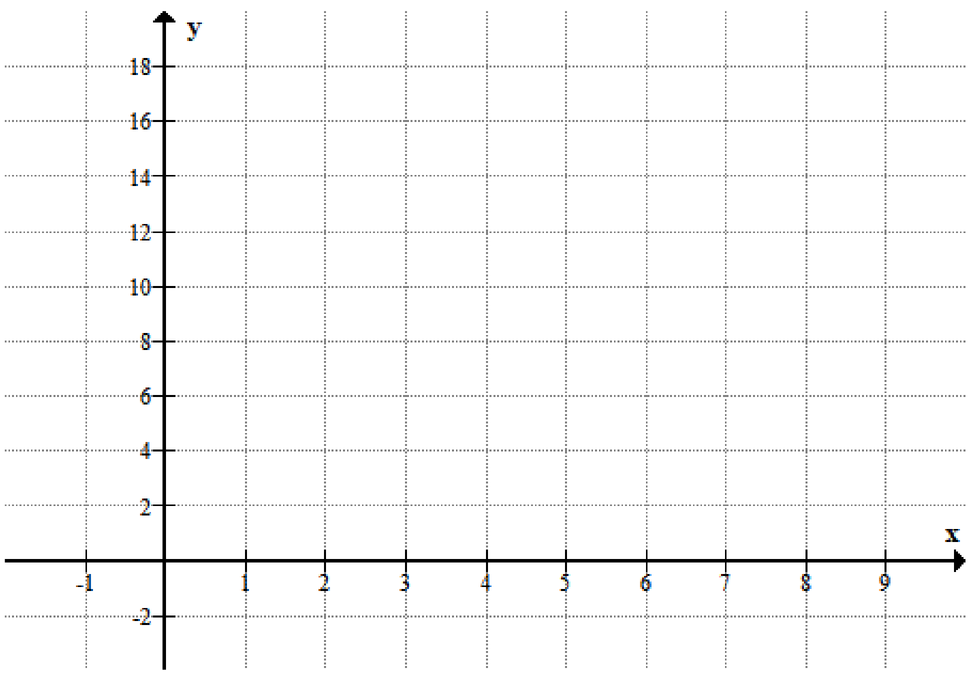
$$f(x) = 5\cos(45x)° + 5\,,\quad 0 \le x \le 8$$



9. [Maximum mark: 4]

Sketch the graph of the function

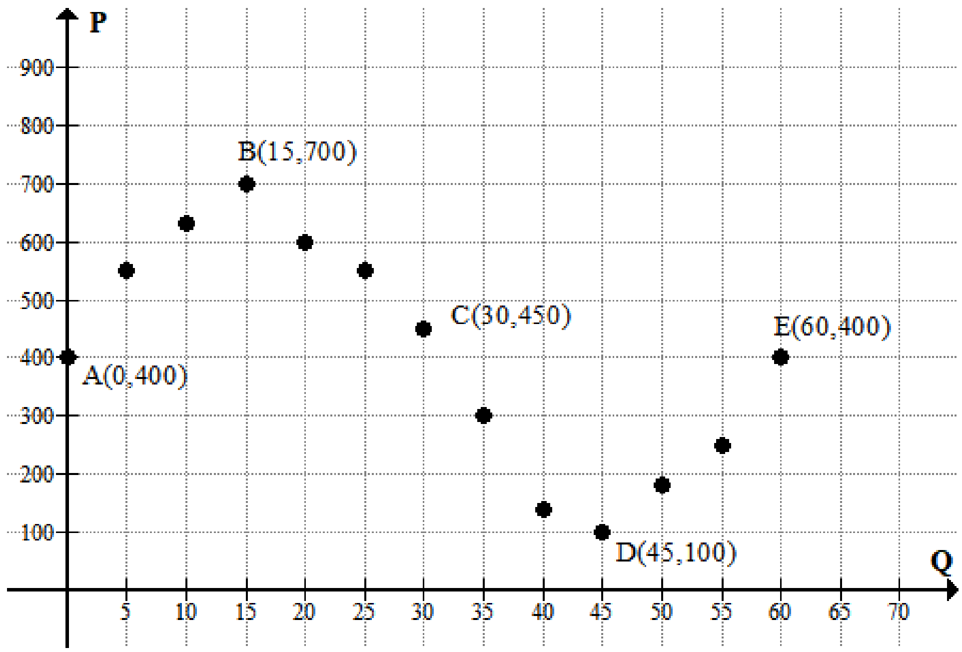
$$f(x) = -6\cos(90x)° + 10\,,\quad 0 \le x \le 8$$



B. Paper 2 questions (LONG)

10. [Maximum mark: 15]

The following diagram shows a point series (P, Q) on the Cartesian plane



Peter believes that the points follow a sinusoidal model.

(a) Use the points A(0,400), B(15,700), D(45,100) and E(60,400) to estimate

(i) the equation of the central (or principal) axis.

(ii) the amplitude

(iii) the period of the model function. [3]

(b) Find a model of the form $P = a\sin(bQ)^{\circ} + c$ for the model series. [4]

(c) On the same diagram above, sketch the graph of your model. [2]

(d) The **exact** value of P at $Q = 30$ is 450 (point C). Write down the estimation of the model for the value of P at $Q = 30$ and **hence** find the percentage error. [3]

(e) Peter needs a prediction for the value of P when $Q = 90$

(i) Find an estimation according to your model.

(ii) Write down a comment about the validity of your estimation. [3]

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11. [Maximum mark: 12]

A spring is suspended from the ceiling. It is pulled down and released, and then oscillates up and down. Its length, l centimetres, is modelled by the function

$$l = 33 + 5\cos\left((720t)^{\circ}\right),$$

where t is time in seconds after release.

(a) Find the length of the spring after 1 second. [2]

(b) Find (i) the minimum length of the spring. (ii) the maximum length of the spring. [3]

(c) Find the first time at which the length is 33 cm. [2]

(d) What is the period of the motion? [2]

(e) Within the first 5 seconds, find the number of times the spring has

(i) its minimum length (ii) a length of 30cm. [3]